High-fidelity adaptive curvelet domain primary-multiple separation

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Abstract
In this paper, we propose an adaptive scheme for primary-multiple separation whereby the multiples are first estimated from the seismic data and then removed using the curvelet transform. Because of the sparseness of seismic data in the curvelet domain, the primary-multiple separation problem is formulated by incorporating L1- and L2-norms, based on the framework of the Bayesian Probability Maximization theory. An iterative soft-thresholding method is used for solving the optimization problem. Prior to removal, the predicted multiples are preconditioned to match the actual multiples in the seismic data by least-squares matched filtering. We show that such an adaptive implementation is more robust and has a superior performance to the conventional least-squares method for attenuating multiples.

To improve the effectiveness of primary-multiple separation for complex data, we develop a frequency-regularized adaptive curvelet domain separation approach. The method is optimized for different frequencies to improve attenuation in the presence of noise and in areas where multiple models are less accurate (e.g. narrowing of frequency bandwidth due to the convolution process in SRME). Accordingly, this extension provides more flexibility and leads to higher separation fidelity than its original form. We demonstrate the application of our approach on synthetic and field data. The results obtained from our approaches show significant improvement over those obtained from conventional least-squares methods.

Introduction
Multiple attenuation plays an important step in the pre-processing of seismic data, and can directly affect the quality of the seismic image. Generally, model-based multiple attenuation involves two steps: firstly, the multiples are predicted and secondly, the predicted multiples are removed from the seismic data leaving the primary reflections. Considerable effort has been devoted to the prediction of multiples in the last two decades. Surface-Related Multiple Elimination (SRME) is used routinely in the industry to eliminate long-period multiples. Short-period multiples generated from the shallow seafloor and internal multiples generated by subsurface interfaces of high impedance contrast have also received attention in the areas of multiple modelling (Hargreaves, 2006; Hung et al., 2010; Wang et al., 2011; Wang et al., 2012; Yang and Hung, 2012).

As well as multiple prediction, an effective strategy for separating multiples from primaries is equally important. One of the most widely accepted separation strategies is the L2-norm based least-squares separation method (LS) (Verschuur and Berkhout, 1997). This method allows for a certain degree of inaccuracy in the prediction of multiples, including traveltime, amplitude and spectrum errors. However, a compromise has to be made between the preservation of primaries and the attenuation of multiples, especially in places where primary and multiple events cross over one another or overlap. Meanwhile, different multiples have different characteristics in terms of their strength and multiple period. Internal multiples tend to have relatively low amplitude and shorter multiple period in comparison with deepwater surface-related multiples (Wang et al., 2012). Hence, it is beneficial to have a separation tool that can be optimized for handling various types of multiples. For this reason, interest in curvelet-based separation methods has increased recently. These methods have the advantage of minimizing the damage to primary events due to the compact nature of the curvelet transform of seismic data (Herrmann et al., 2008). The various curvelet domain separation approaches that have been developed so far can be categorized into non-adaptive and adaptive implementations. On the one hand, non-adaptive implementations may encounter numerical divergence if predicted multiples deviate from multiples in the data (Herrmann et al., 2007; Saab et al., 2007); conversely, current adaptive implementations are more tolerant to deviation of predicted multiple models from actual multiples. However, existing adaptive approaches are either only correct for limited misalignment between predicted and actual multiples, or are affected by high computational cost due to the use of curvelet matched filtering (Herrmann et al., 2008; Neelamani et al., 2010).

In this paper, we present our adaptive approaches for curvelet domain primary-multiple separation. Our approaches

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aim to improve the robustness and fidelity of current implementations of curvelet domain primary-multiple separation, and overcome the difficulties that are not fully handled by LS methods. In the following section, we first introduce the concept and the characteristics of the curvelet transform, and its application in primary-multiple separation. We then propose the theoretical framework and the primary-multiple separation workflow. In the following sections, we demonstrate the proposed approaches by testing on synthetic and field examples. Finally, we summarize and recommend these approaches for addressing the primary-multiple separation problem in the future.

Framework of curvelet domain separation

Curvelet domain separation for removing multiples from noisy seismic data involves transforming seismic data into the curvelet domain and a process for simultaneously separating multiples and primaries in the curvelet domain. The curvelet transform is a multi-scale and multi-dimensional transform (Candès et al., 2006), which can be written as:

$$C(j,\bar{k},l)=\int_{R^2} D(t,x)\,\varphi_{j,\bar{k},l}(t,x)\,dtdx \quad (1)$$

where $C(j,\bar{k},l)$ is the curvelet coefficient indexed by its frequency band *j*, dip *l* and time-space displacement $\bar{k}$, and *D(t,x)* is the 2D seismic sample at time *t* and position *x;* $\varphi_{j,\bar{k},l}(t,x)$ is the curvelet basis. Both $\bar{k}$ and *l* increase in dyadic order for every other *j*, hence the term 'multi-scale'. In contrast to the time-space or frequency basis, a curvelet is localized in both frequency and time-space, as shown in Figure 1 (a-b). By varying its indices, the scale, the dip and the spatial location of the curvelets will change accordingly (Figure 1 (c-f)). In seismic data, most events are either linear or curved in shape within a small spatiotemporal window; hence the needle-like curvelets form a suitable and natural basis for representing seismic data. This makes events that are widely spread out in the spatiotemporal domain very compact in the curvelet domain with a few coefficients. This leads to the sparseness of seismic data in the curvelet domain. There exists a way to exploit this sparsity for separating multiples from primaries by using Bayesian Probability Maximization (BPM) (Saab et al., 2007). The probability of predicted primary and multiple can be written as:

$$P\left(\tilde{P},\tilde{M}|D,M\right)=\frac{P\left(\tilde{P},\tilde{M}\right)P\left(D|\tilde{P},\tilde{M}\right)P\left(M|D,\tilde{P},\tilde{M}\right)}{P\left(D,M\right)} \quad (2)$$

where $P(*)$ denotes the probability of the variables in the brackets; '|' denotes the condition sign, and the variables on its right are conditions. $\tilde{P}$ and $\tilde{M}$ are the predicted primary and multiples, and *D* and *M* are the known input data and multiple model. BPM yields the maximization of the joint probability of $\tilde{P}$ and $\tilde{M}$ for given *D* and *M*, i.e., $P\left(\tilde{P},\tilde{M}|D,M\right)$. The conventional implementation of BPM is equivalent to the LS method. This is because the prior probability distribution of data and model is preset to Gaussian; and by solving BPM, it extracts the power indices of the distribution function to formulate a quadratic summation form. In the curvelet domain, the L1-norm was introduced in the optimization problem since the distribution of the sparse coefficients is asymptotically closer to a Laplacian than to a Gaussian function. An iterative soft-thresholding algorithm was applied to solve this optimization problem (Daubechies et al., 2004).

In this paper, noting that the convergence of the iterative solver used by Saab et al. (2007) relies on an initial estimate of the predicted multiples that is sufficiently close to the actual multiples in the data, we design the least-squares matching filtering in the optimization process to bring the amplitude, traveltime and spectrum of the predicted model

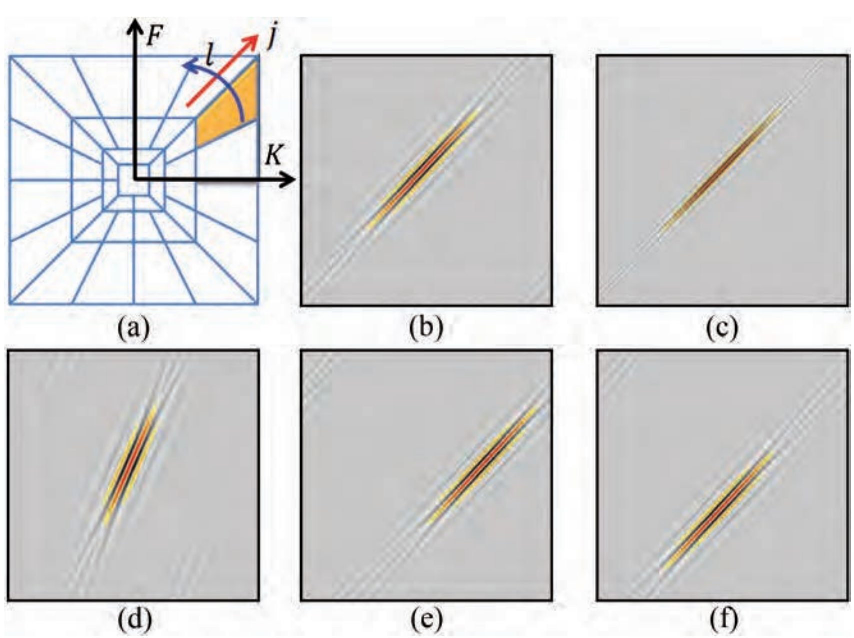

Figure 1 *(a) Curvelet tiling in the frequency domain; the red and blue arrows denote the ascending direction of index j and l, respectively. (b) the time-space presentation of the curvelet, annotated with an orange colour in (a) and vanishing k1 and k2; (c-f) the curvelets in time-space domain with larger values of j, l, k1 and k2, respectively.*

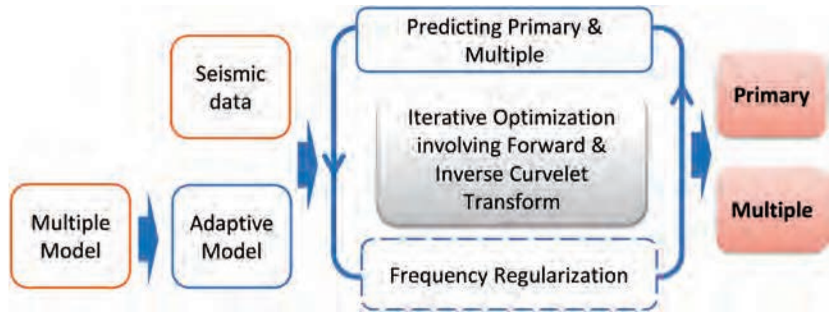

***Figure 2** Flowchart of ACDS and FrACDS. The 'Frequency-regularization' step in the dashed box is only applicable to FrACDS.*

closer to those of the actual multiples. Such preconditioning of the model is likely to make the initial solution fall into the convergence range of the optimization solution. In contrast to the standard LS method that often makes a compromise between primary preservation and multiple attenuation, preconditioning the multiple model minimizes damage to the primary events, while matching the predicted multiples to the actual multiples in the data. Our implementation is to replace the original predicted model M' in Saab's equation by its adapted version by applying the matching filter $\hat{f}_{LS}$ to M', as shown in Eq. (3.1), where. $\hat{f}_{LS}$ is designed by Eq. (3.2) prior to solving the optimization problem Eq. (3.1).

$$f\left(\hat{P}_c,\hat{M}_c\right)=\min\left\{\left\|P_c\right\|_{1,w_1}+\left\|M_c\right\|_{1,w_2}+\left\|C^{-1}M_c-\hat{f}_{LS}*M'\right\|_2^2+\eta\left\|C^{-1}\left(P_c+M_c\right)-D\right\|_2^2\right\} \quad (3.1)$$

$$g\left(\hat{f}_{LS}\right)=\min\left\|D-f_{LS}*M'\right\|_2 \quad (3.2)$$

In Eq. (3.1), P_c and M_c denote the primaries and multiples in the curvelet domain; D and M' are the data and the predicted multiple model in the time-space domain, respectively C denotes the forward curvelet transform and C^{-1} the inverse curvelet transform. Subscripts '1, w_1' and '1, w_2' denote the weighted L1 norms, with weights w_1 and w_2 being proportional to the curvelet coefficients of the initial estimation of the model $C\left(\hat{f}_{LS}*M'\right)$ and the primary $C\left(D-\hat{f}_{LS}*M'\right)$, respectively; subscript '2' denotes the L2-norm. Weights $w_1$and w_2 in the terms of L1-norm penalize the orthogonality of primaries and multiples in their curvelet domain (Saab et al., 2007). Parameter η is inversely proportional to the estimated overall noise level. For solving equation (3), the aforementioned iterative soft-thresholding algorithm can still be applied. We term this implementation as the Adaptive Curvelet Domain Separation (ACDS) approach.

One of the advantages of decomposing seismic data in the curvelet domain is that the basic functions have different frequency content. We can make use of this feature by extending ACDS to be optimized for different frequency bands. Applying the same parameters across all frequencies in ACDS may not be effective for all types of complex data since the level of noise and signal may vary across frequencies, and the degree of discrepancy between the predicted and actual multiples (e.g. narrowing of bandwidth in the predicted model caused by convolution in SRME) is frequency-dependent. Since the curvelet transform naturally partitions data into different frequency bands, it is feasible to manipulate the curvelets in each frequency band independently. We therefore propose a new approach, referred to as Frequency-regularized Adaptive Curvelet Domain Separation (FrACDS). With FrACDS, the objective function of the optimization problem $F\ (P_c, M_c)$ can now be recast as:

$$F\left(P_c,M_c\right)=\sum_j f_j\left(P_c,M_c\right) \quad (4)$$

where $f_j\ (P_c, M_c)$ holds the same expression as Eq. (3) except that the controlling parameters depend on scale j. The optimization parameters for different scales are determined by the mismatch of the multiple models from the actual multiples within this scale. The flowchart of the overall process is shown in Figure 2. FrACDS provides more flexibility to effectively separate primaries and multiples in the presence of model inaccuracy and noise contamination for each frequency band.

Synthetic and field data examples

Two simple synthetic examples, shown in Figure 3, were first tested to assess the performance of our proposed approaches. Without loss of generality, we designed the multiple event to be linear and curved in the respective examples (Figure 3 (a2) and (b2)), and the multiple events cross over the horizontal primary events (Figure 3 (a1) and (b1)). To simulate the practical situation, a moderate mismatch of the frequency spectra and the dips between the multiple models and actual multiples presented in the data were included. The conventional LS approach leaves residual multiples at the crossings, as annotated by blue arrows in Figure 3 (a3) and (b3). The wavelet of the primary is distorted after applying LS for primary-multiple separation. The difficulty is caused by the crossing of two multiple events and one primary event in example (a) and the overlapping of the multiple and primary in example (b), and by the mismatch of dip and wavelet between the models and the multiples. The multiples in example (a) was dramatically cleaned up by a non-adaptive curvelet domain separation method, but the apparent remnant of the steeper multiple event is still observable (Figure 3 (a4)). This is due to different degrees of mismatch between the two modelled and the actual multiple events. In contrast, they are more cleanly removed with minimal damage to the

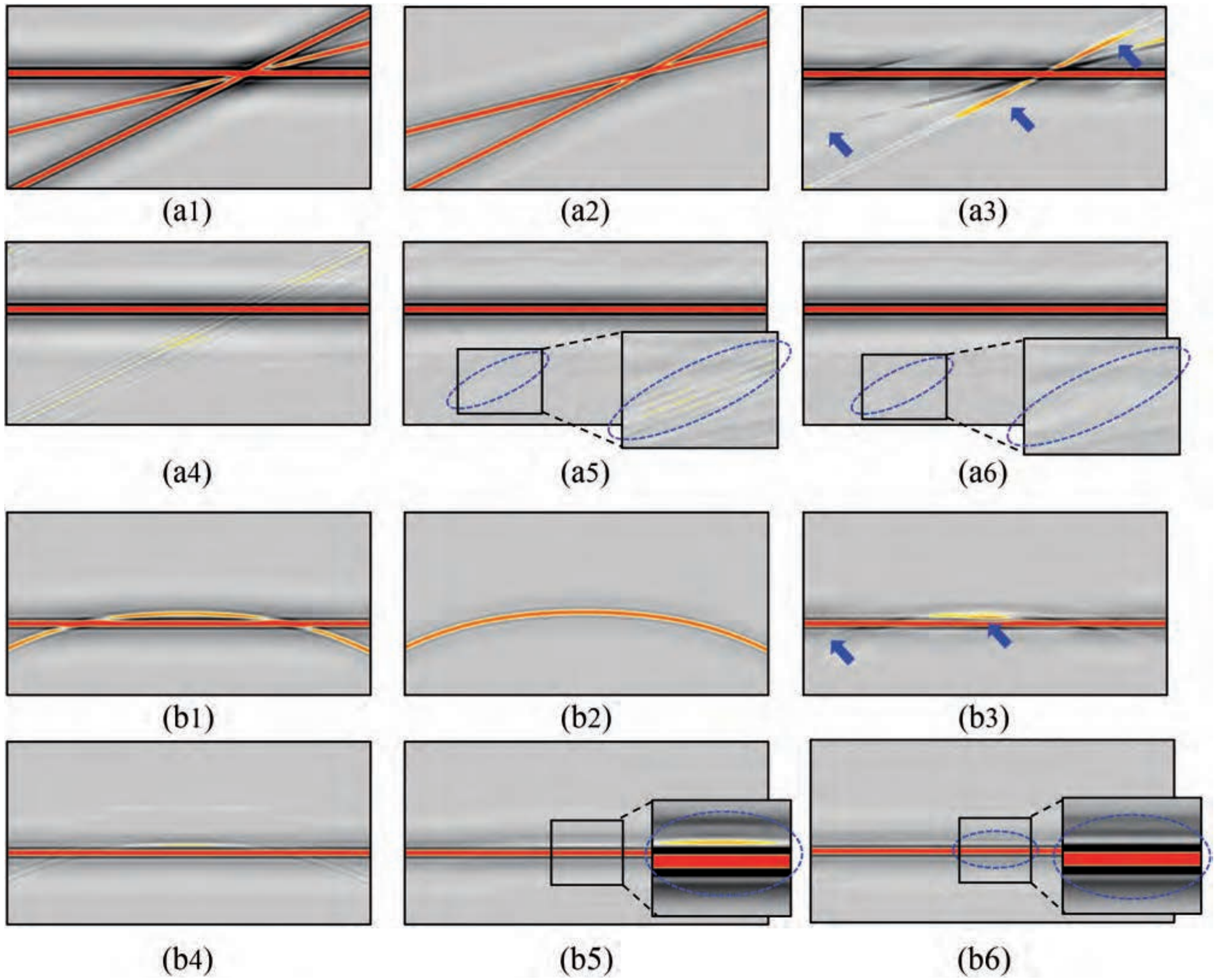

Figure 3 *Upper panels (a) and lower panels (b) show two synthetic examples illustrating the effects of primary-multiple separation by LS, ACDS and FrACDS approaches. Panel (1): multiple contaminated data; (2): multiple models; (3): optimal LS results; (4): non-adaptive curvelet domain separation results; (5): ACDS results; (6): FrACDS results. The zoom-in insets of (5) and (6) with +6 dB gain highlight the difference between the ACDS and FrACDS results, as annotated by the blue circles.*

primaries by ACDS (Figure 3 (a5)). This is because primary and multiple events at crossings are represented by different curvelet coefficients and the adaptation of the model brings the mismatch level closer between the two multiple events and their corresponding models. Moreover, for example (b), the non-adaptive approach cannot completely eliminate the multiple, which is affected by the discrepancy of frequency spectra and amplitude between the actual curved multiple and its model (Figure 3 (b4)). This issue is well resolved by ACDS as most multiple energy is properly removed, except for the overlapped area in (b5). However, we realize that multiple models in example (a) and (b) differ from the actual multiples in the high- and low-frequency range, respectively; and there are still observable residuals of high-frequency multiples indicated in Figure 3 (a5) and low-frequency multiples in (b5). The strategy in applying FrACDS for primary-multiple separation is to put special emphasis on model fidelity in different frequency bands so as to more effectively remove multiple-related curvelet coefficients in the corresponding frequency components. Consequently, the separation results are improved by FrACDS with much less residual multiple energy than by LS and ACDS (Figure 3 (a6)). The insets annotated by the blue circles highlight the difference between results generated by ACDS, (a5) and (b5), and FrACDS, (a6) and (b6). It is clear from these examples that FrACDS suppresses the high- and low-frequency multiples in an optimal way. In addition, the damage to primary energy, especially in the primary-dominant frequency band, is minimized by FrACDS. It can be seen that FrACDS provides fidelity of primary-multiple separation in seismic data.

We tested the primary-multiple separation approaches on the 2004 BP 2D model shown in Figure 4. We applied LS and FrACDS to remove the multiples predicted by reverse time demigration (Billette and Brandsberg-Dahl, 2005; Zhang and Duan, 2012), and compared the reverse time migration (RTM) stacks. From the comparison, it is obvious that the first-order water bottom multiple is completely removed by FrACDS but not by LS, annotated by the blue arrows (Figure 4 (b) and (c)). The boundaries of salt bodies are preserved by FrACDS, but are severely contaminated by LS, as indicated by the yellow arrows in (e) and (f); besides, the migration swings are attenuated at the top of the salt body on the right with FrACDS. The superior preservation of primary events by FrACDS is also evident as shown in the areas of the anomaly (lower-right) and the parallel sedimentary (upper-left) areas, indicated by the cyan arrows. Apparently, from panel (e) and (f) showing the attenuated energy, FrACDS outperforms LS in terms of both primary preservation and multiple attenuation.

The field data that we used for testing our approach is from a 3D BroadSeis survey conducted by CGG for Shell Brunei in 2012, which covers an area of 3100 km^2 (Soubaras and Dowle, 2010). 3D SRME was applied to predict surface-related multiples for this project (Lin et al., 2005). Noticing that the multiple model has a certain degree of discrepancy in terms of frequency content and the data are contaminated with high-frequency noise, we applied FrACDS to provide more flexibility in solving these difficulties. In Figure 5, common mid-point (CMP) gathers are shown after normal moveout (NMO) correction. Multiples with a relatively low amplitude level are masked by the strong primaries, as highlighted by the green dashed box in Figure 5(b) and 5(c). The LS approach removes some of the overlapping primary energy indicated by the red ellipses shown in Figure 5(e). On the contrary, FrACDS removes the multiples with minimal damage to the primaries, as highlighted by the red ellipses in Figure 5(f). This is because the dip differentiation between the overlapping primaries and multiples was captured by FrACDS, which enhances their separability in the curvelet domain. Furthermore, due to the presence of the high-frequency noise, the multiple model in the corresponding high-frequency bands is less credible than its low-frequency component. The LS approach fails to cleanly remove the high-frequency multiples, as is manifested by the amount of multiples removed shown in (e); however, FrACDS fully recognizes the model fidelity and noise level across all frequencies, thus the separation result is not compromised in different frequency bands (Figure 5(f)).

The benefit of multiple attenuation by FrACDS can also be seen in pre-migration stacks (Figure 6). Again, it is evident that the FrACDS approach preserves primaries better than the LS method. This is shown by the better continuity of the strong primaries in the highlighted area of Figure 6 (c) compared to Figure 6 (b), and a more contrasted comparison of the difference between input seismic data and resulting primary in Figure 6 (e) and (f). Meanwhile, the higher resolution of detailed structures in the removed multiples in (f) than in (e) also indicates the high-frequency multiple residuals in the LS result depicted in (b), but not in the FrACDS result shown in

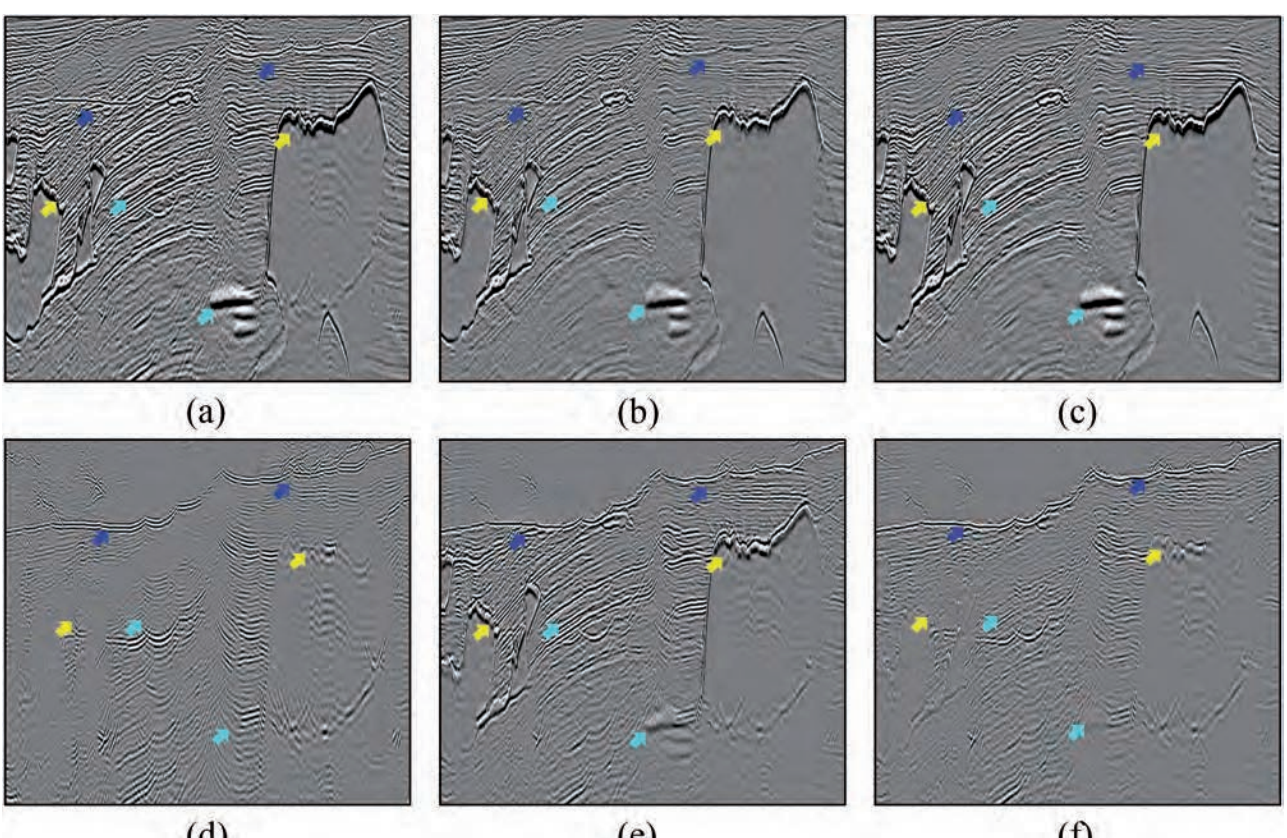

***Figure 4** (a) multiple contaminated RTM stack of BP2004 synthetic model. (b) RTM stack with multiple attenuated by LS; (c) by FrACDS; (d) RTM stack of RTDM generated multiple model; (e): (a)-(b); (f): (a)-(c).*

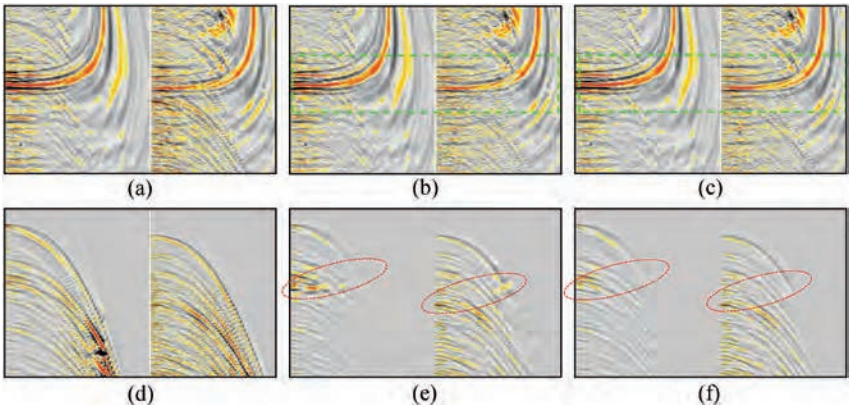

Figure 5 *NMO-corrected CMP gathers. (a) input data before multiple attenuation; (b) multiple separation result by LS; (c) multiple separation result by FrACDS; (d) 3D SRME multiple model; (e) difference between (a) and (b); (f): difference between (a) and (c).*

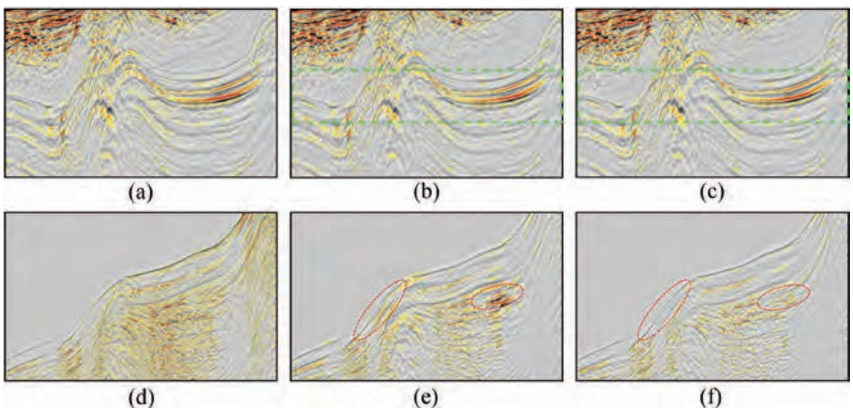

Figure 6 *Pre-migration full stacks. (a) input data before multiple attenuation; (b) multiple separation result by LS; (c) multiple separation result by FrACDS; (d) 3D SRME multiple model; (e) difference between (a) and (b); (f): difference between (a) and (c).*

(c). In this example, FrACDS is used to overcome the narrowing of the frequency bandwidth caused by the convolution process in 3D SRME and for handling the high-frequency noise so that it can be more effective in separating the multiple-related curvelet coefficients from the primary-related curvelet coefficients. Consequently, the separation result by FrACDS presents a cleaner image with a lower noise level and less residual multiples compared to the LS result; while the damage of the primary energy in the low-frequency band is minimized, as indicated by the arrows and dashed circles in Figure 6. FrACDS honours differentiation of local dips between primaries and multiples, and allows for a higher degree of model deviation; these aspects allow FrACDS to produce favourable results. Apart from protection of the low-frequency components, FrACDS is very robust in attenuating high-frequency multiples, in the presence of moderate noise levels. Hence, we have demonstrated that FrACDS provides a high-fidelity solution for primary-multiple separation of field data.

Conclusion

We have developed a modified approach of primary-multiple separation for removing multiples from noisy seismic data in the curvelet domain. Synthetic and field data examples demonstrate that our approaches outperform the conventional LS method in terms of multiple removal and primary preservation (and to some extent, noise attenuation). Based on current implementations of the curvelet domain separation method as a starting point, we provide an alternative by adaptively preconditioning the predicted multiple models using least squares matched filtering. Our approach has been demonstrated to be an effective and robust tool for handling various types of multiple models. To meet the challenges caused by to the increasing complexity of seismic data, we have also proposed a more flexible implementation of primary-multiple separation in the curvelet domain, FrACDS, by incorporating optimization for different frequency bands. FrACDS provides optimized separation results especially in the presence of high noise levels and tends to preserve primary events better by reducing the difference between predicted and actual multiples. The generality of FrACDS allows it to be effectively applied for handling internal multiples as well as surface-related multiples and on marine data as well as land data. While our current implementation is in 2D, the algorithm that we have described can readily be extended to 3D. We envisage that the 3D implementation will further enhance the effectiveness of primary-multiple separation, particularly when multiples cross over or overlap with primaries by having an extra dimension to differentiate multiples from primaries.

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